Moments

k^{th} moment of a data set (total n observations) about an arbitrary point A is given by

$$\mu_k' = \sum (x - A)^k / n$$

If A is the arithmetic mean, it is known as the central moment and is represented by μ_k.

The first central moment (μ_1) is always equal to 0. Second central moment (μ_2) is the variance, third central moment (μ_3) indicates skewness of a distribution and the fourth central moment (μ_4) is an indicator of kurtosis.

Skewness

If a data set is uniform around its mean, it is called symmetric. For a symmetric distribution, Arithmetic Mean = Median = Mode.

If the data has a tail on the positive side (or higher than mean), it is called a positively skewed distribution. In this case, Arithmetic Mean > Median > Mode.

If the data has a tail on the negative side (or less than mean), it is called a negatively skewed distribution. In this case, Arithmetic Mean < Median < Mode.

The following measures are used for skewness.

1. Karl Pearson's γ_1 and β_1 coefficients ($\gamma_1 = \sqrt{\beta_1} = \mu_3 / \sigma^3$)
2. Co-efficient of skewness = (Arithmetic Mean - Mode) / σ or 3 * (Arithmetic Mean - Median) / σ
3. Bowley's Co-efficient of skewness = ($Q_1 + Q_3 - 2M_e$) / ($Q_3 - Q_1$)

Kurtosis

Kurtosis is used to measure the peakedness of a distribution. It is measured with Karl Pearson's γ_2 and β_2 coefficients. It can be of the following types:

1. **Leptokurtic:** Distribution is tall, more values near the average, $\gamma_2 > 0$
2. **Mesokurtic:** Normal distribution , $\gamma_2 = 0$
3. **Platykurtic:** Distribution is flat, $\gamma_2 < 0$