Confidence Intervals to Assess Variation in Fat Content at a Fast-Food Restaurant

Hamburrgerr, Inc. is a fast-food restaurant serving hamburgers among a few other items. The restaurant claims that the average fat content in the hamburgers is 15 grams.

Section 3.1 gives a brief description of the define phase. Section 3.2 illustrates the measure phase with detailed instructions for using Minitab®. The analyze phase is briefly discussed in Section 3.3. Section 3.4 illustrates the improve phase with detailed instructions for using Minitab®. Finally, the control phase is briefly discussed in Section 3.5.

3.1 Define Phase

A few of the customers complained to the operations manager recently that the fat content in the hamburgers appeared to be higher than the restaurant's claim of 15 grams. The operations manager wishes to use 95% confidence intervals to verify whether the restaurant's claim of 15 grams (average) of fat in the hamburgers is correct. He also wishes to verify his assumption that the standard deviation of the fat content is less than 1 gram.

3.2 Measure Phase

The operations manager randomly selects 20 hamburgers and measures the fat content (in grams) in each of them as follows: 15.5, 12.3, 15.4, 16.5, 15.9, 17.1, 16.9, 14.3, 19.1, 18.2, 18.5, 16.3, 20.0, 19.0, 15.6, 13.5, 14.0, 16.5, 19.0, and 18.6.

Before constructing a confidence
interval for the above data, it is important to check whether the data are in statistical control. Because each number is for one hamburger, the appropriate set of control charts is I-MR (individual and moving range) charts. Figure 3.1 shows how to select I-MR charts in Minitab®. Doing so will open

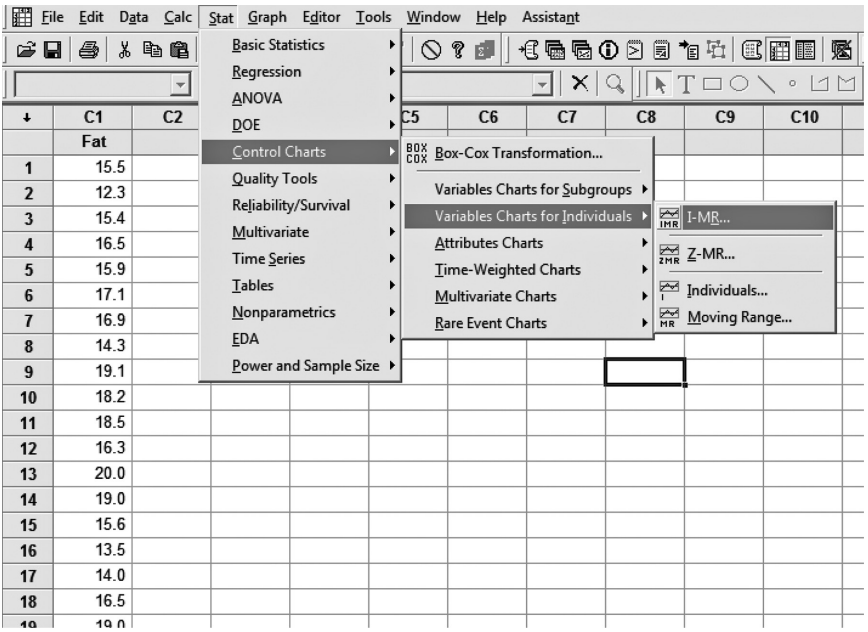

FIGURE 3.1
Selection of "I-MR Chart" before improvement.

the dialog box shown in Figure 3.2. Select "Fat" for "Variable", and click on "OK." The I-MR charts shown in Figure 3.3 will be the result. As is evident, the data are in statistical process control.

Now that the data are in statistical control, the operations manager wishes to construct the 95% confidence interval for the mean of fat content in

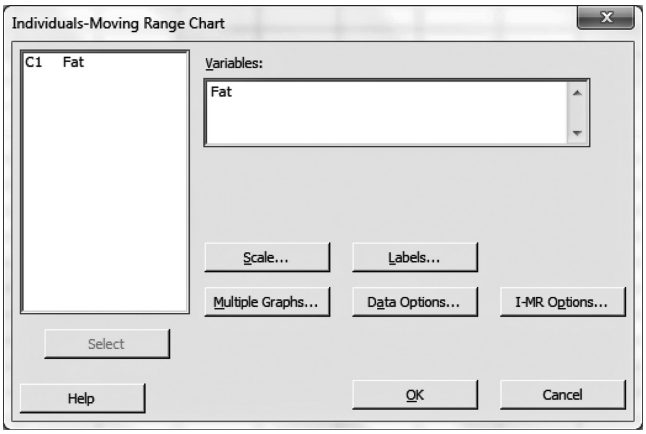

FIGURE 3.2
Variable of "Fat" for I-MR chart before improvement.

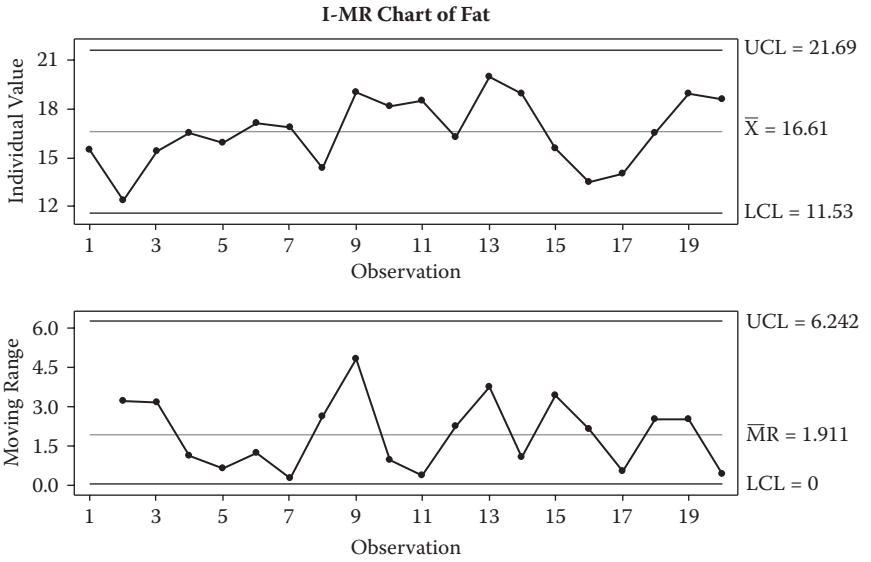

FIGURE 3.3
I-MR charts before improvement.

population of the hamburgers. He also wishes to check whether the data are normally distributed before relying on the confidence interval. The "Graphical Summary" option in Minitab® produces both the normality test result as well as the confidence interval. Figure 3.4 shows to how to select the graphical summary. Doing so will open the dialog box shown in Figure 3.5. Select "Fat" for "Variable" and click on "OK". The graphical summary shown in Figure 3.6 will be the result. Because the *P*-value (0.624) of the Anderson–Darling normality test is greater than 0.05, the data are normally distributed. Notice that the population mean is estimated to be between 15.625 grams and 17.595 grams. The entire confidence interval (15.625, 17.595) is greater than the claim of 15 grams, therefore the operations manager considers this a serious issue.

The operations manager then proceeds to verify his assumption that the standard deviation of the fat content is less than 1 gram. Figure 3.7 shows how to select "1 Variance" in Minitab®. Doing so will open the dialog box shown in Figure 3.8. Select "Samples in columns" for "Data" and "Fat" for "columns". Click on "Options" and it opens the dialog box shown in Figure 3.9. Select "less than" for "Alternative" because the operations manager is interested in the standard deviation being less than 1 gram. Click on "OK" and it takes you back to the dialog box shown in Figure 3.8. Click on "OK" and the output shown in Figure 3.10 is the result. Because the data are normally distributed, we must look at the upper bound (2.89 grams) of the standard deviation given by the chi-square method. This means that 95% of

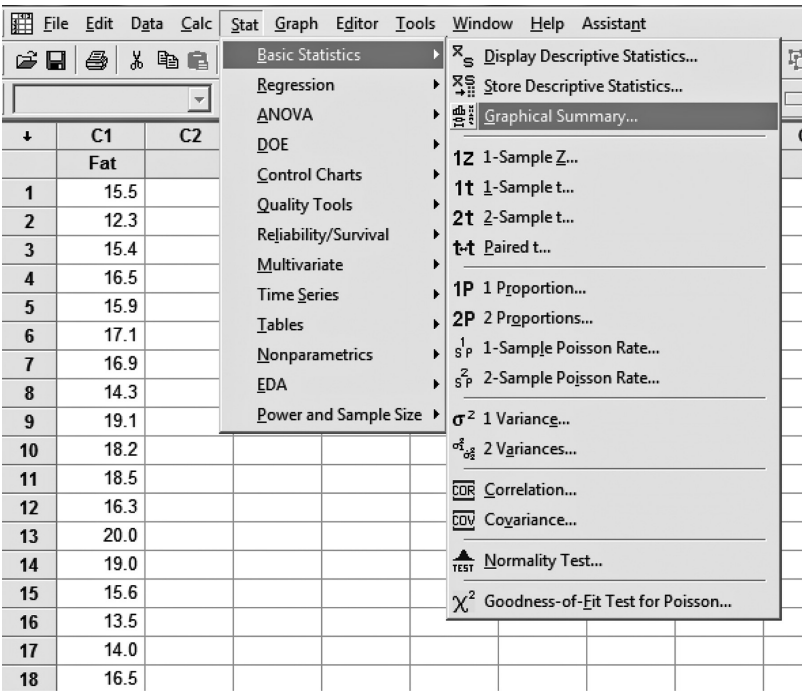

FIGURE 3.4
Selection of "Graphical Summary" before improvement.

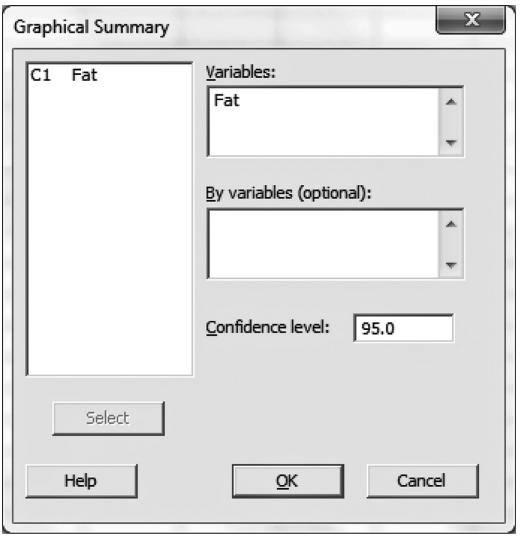

FIGURE 3.5
Variable of "Fat" for graphical summary before improvement.

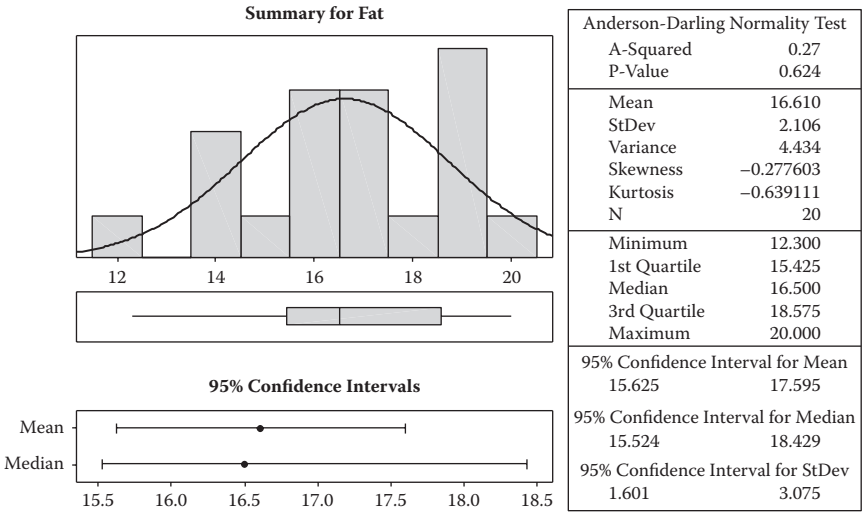

FIGURE 3.6
Graphical summary before improvement.

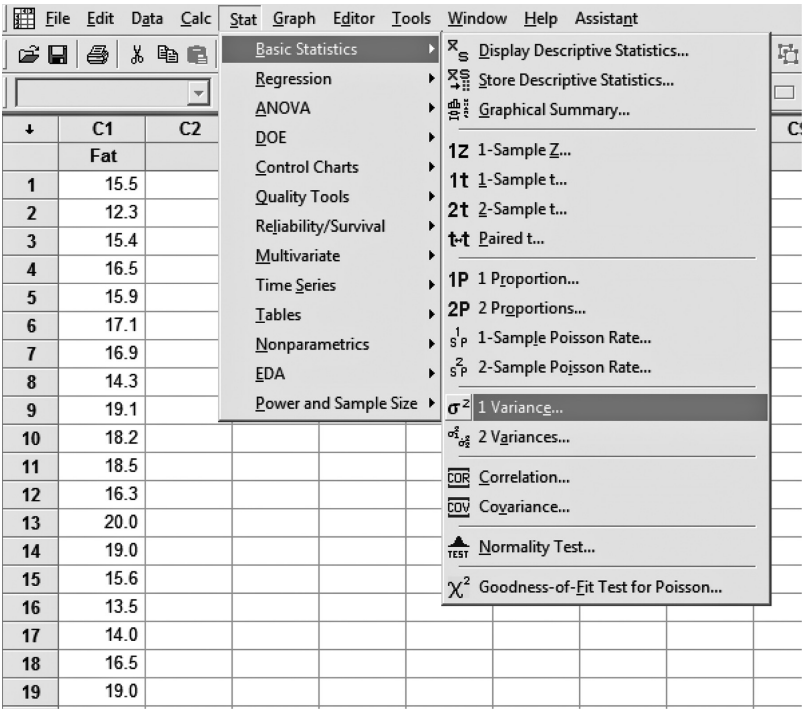

FIGURE 3.7
Selection of "1 Variance" before improvement.

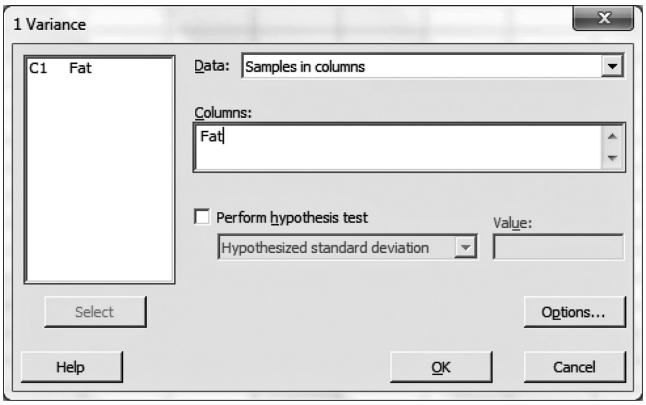

FIGURE 3.8
Selection of "Fat" variable before improvement.

the hamburgers have fat content with a standard deviation that is lower than 2.89 grams. Inasmuch as 2.89 grams is much greater than the assumption of 1 gram, the operations manager considers this also a serious issue.

Before proceeding to the improve phase, the operations manager wishes to check how many of the customers are dissatisfied. To this end, he randomly selects 1,000 customers who bought hamburgers and finds that 83 of them are dissatisfied with the food quality. He then proceeds to construct a confidence interval for the proportion of all customers who are dissatisfied. Figure 3.11 shows how to select "1 Proportion" in Minitab®. Doing so will open the dialog box shown in Figure 3.12. Enter "83" for "Number of events" and "1000" for "Number of trials". Click on "Options" and the dialog box shown in Figure 3.13 opens. Select "less than" for "Alternative" because the operations manager is interested in lowering the proportion of all customers who are dissatisfied. Also, check the box for "Use test and interval based on normal distribution". Click on "OK" and it takes you back to the dialog box shown in Figure 3.12. Click on "OK" and the output shown in Figure 3.14 is

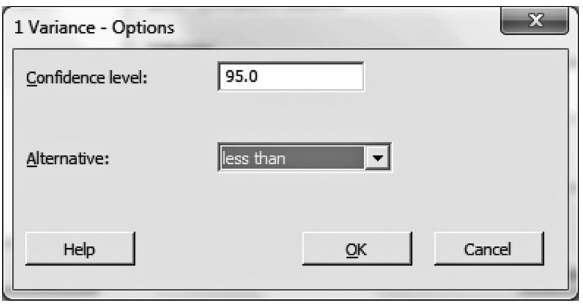

FIGURE 3.9
Selection of "less than" option for estimating standard deviation before improvement.

Test and CI for One Variance: Fat

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Method
The chi-square method is only for the normal distribution.
The Bonett method is for any continuous distribution.

Statistics
Variable   N  StDev  Variance
Fat       20   2.11      4.43


95% One-Sided Confidence Intervals

                       Upper
                       Bound
                         for   Upper Bound
Variable  Method      StDev  for Variance
Fat       Chi-Square   2.89          8.33
          Bonett       2.81          7.92
```

FIGURE 3.10
One-sided 95% confidence interval for standard deviation before improvement.

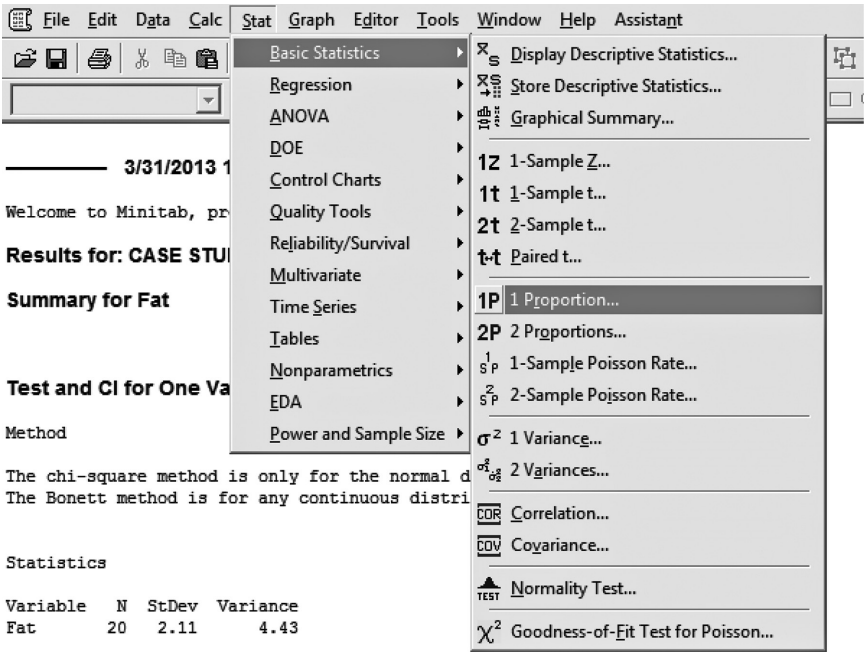

FIGURE 3.11
Selection of "1 Proportion" before improvement.

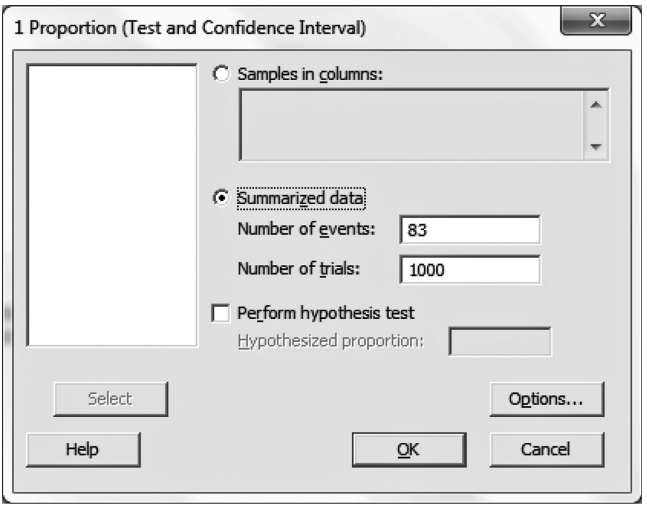

FIGURE 3.12
Entry of "Number of events" and "trials" before improvement.

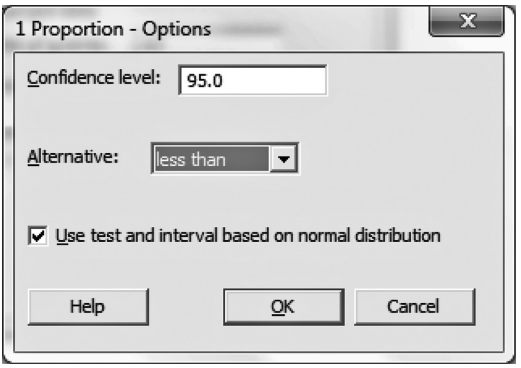

FIGURE 3.13
Selection of "less than" option for estimating population proportion before improvement.

Test and CI for One Proportion

Sample	X	N	Sample p	95% Upper Bound
1	83	1000	0.083000	0.097350

Using the normal approximation.

FIGURE 3.14
One-sided 95% confidence interval for population proportion before improvement.

the result. It is clear that that there is a 95% probability that the proportion of all customers who are dissatisfied is less than 0.097 (or 9.7%). The operations manager wishes to lower this upper bound estimate.

3.3 Analyze Phase

The operations manager analyzes the process and discovers that the fat content and its variation are highly affected by the amount of oil used by the employees on the three different grills used to make hamburgers.

3.4 Improve Phase

The operations manager standardizes the process so that the use of oil on the three different grills is controlled. An automatic machine is bought and installed to dispense oil on a grill each time a hamburger is made.

In order to check whether the process has really improved, the operations manager randomly selects 20 hamburgers and measures the fat content (in grams) in each of them as follows: 14.9, 15.0, 15.4, 15.3, 15.2, 15.1, 14.9, 14.8, 15.6, 14.5, 15.3, 15.8, 15.0, 15.0, 14.3, 15.3, 15.2, 14.7, 15.1, and 14.7.

Open the CHAPTER_3_AFTER.MTW worksheet that has the above data (the worksheet is available at the publisher's website; the data from the worksheet are also provided in the Appendix). Before constructing a confidence interval for the above data, it is important to check whether the data are in statistical control. Because each number is for one hamburger, the appropriate set of control charts is I-MR (individual moving range) charts. The I-MR charts for the above data are shown in Figure 3.15. As is evident, the data are in statistical process control.

Now that the data are in statistical control, the operations manager wishes to construct the 95% confidence interval for the mean of fat content in the population of hamburgers. He also wishes to check whether the data are normally distributed before relying on the confidence interval. Figure 3.16 shows the graphical summary. Because the *P*-value (0.925) of the Anderson–Darling normality test is greater than 0.05, the data are normally distributed. Notice that the population mean is estimated to be between 14.887 grams and 15.223 grams. Inasmuch as the claim of 15 grams is within the interval (14.887, 15.223), it is clear that the process has improved with respect to the mean fat content.

The operations manager then proceeds to verify his assumption that the standard deviation of the fat content is less than 1 gram. As is clear from the

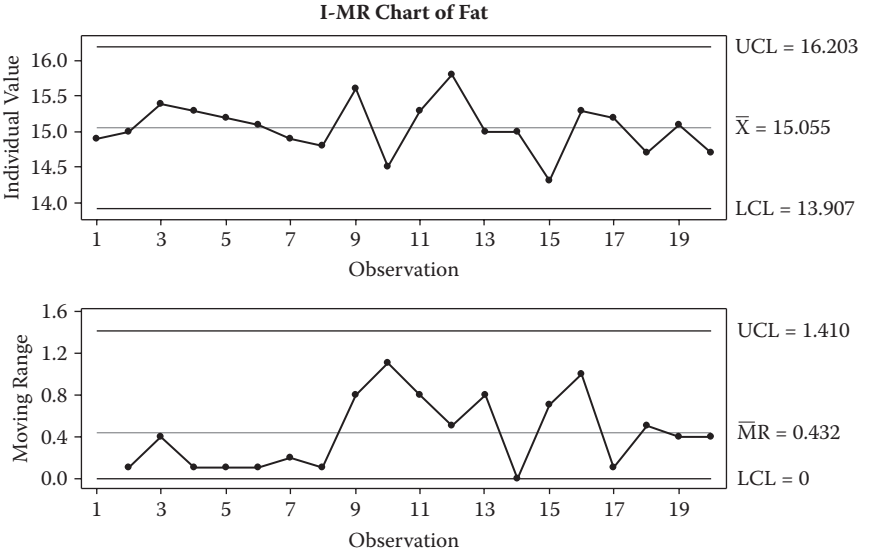

FIGURE 3.15
I-MR charts after improvement.

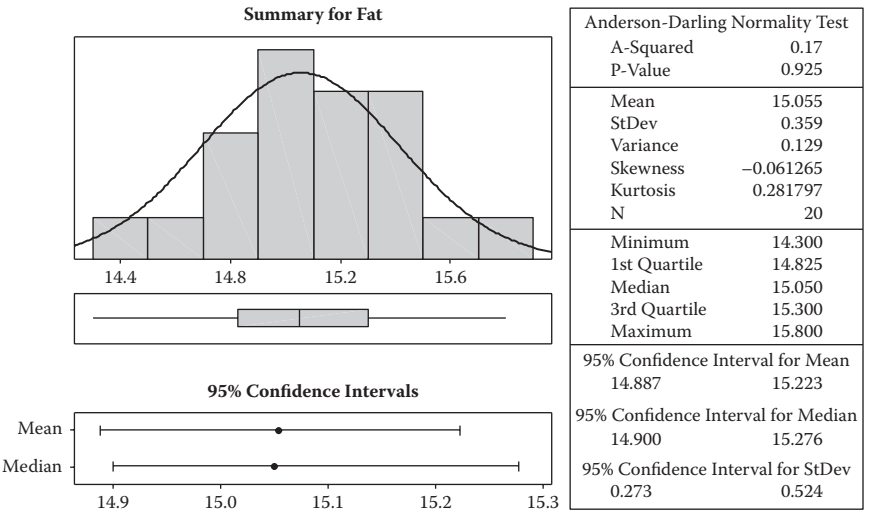

FIGURE 3.16
Graphical summary after improvement.

Test and CI for One Variance: Fat

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Method

The chi-square method is only for the normal distribution.
The Bonett method is for any continuous distribution.


Statistics

Variable   N  StDev  Variance
Fat       20  0.359     0.129


95% One-Sided Confidence Intervals

                        Upper Bound   Upper Bound
Variable  Method          for StDev  for Variance
Fat       Chi-Square          0.492         0.242
          Bonett              0.504         0.254
```

FIGURE 3.17
One-sided 95% confidence interval for standard deviation after improvement.

output shown in Figure 3.17, the upper bound is 0.492 gram for the standard deviation (given by the chi-square method). This means that 95% of the hamburgers have fat content with a standard deviation that is lower than 0.492 gram. Because 0.492 gram is less than the assumption of 1 gram, it is evident that the process has improved with respect to the standard deviation as well.

One can also simulate confidence intervals using Minitab®. The operations manager wishes to perform the simulation for 10 normally distributed samples with a sample size of 20 hamburgers. Doing what is shown in Figure 3.18 opens the dialog box shown in Figure 3.19. Select "20" for "Number of rows of data to generate". Enter "C3-C12" for "Store in column(s)". Because, after process improvement, the population mean is approximately 15 grams and the population standard deviation is approximately 0.5 grams, enter "15" for "mean" and "0.5" for "Standard deviation". Click on "OK" and the data shown in Figure 3.20 are the result.

Before proceeding to simulation, it is a good idea to check whether the data generated are in statistical control. The sample size is greater than 10, therefore the appropriate control charts are Xbar-S charts (sample means and sample standard deviations). Figure 3.21 shows how to select "Xbar-S". Doing so will open the dialog box shown in Figure 3.22. Select "Observations for a subgroup are in one row of columns" from the drop-down menu, and select "C3-C12" in the empty box below the menu. Click on "OK" and the Xbar-S charts shown in Figure 3.23 are the result. It is clear that the data are in statistical control.

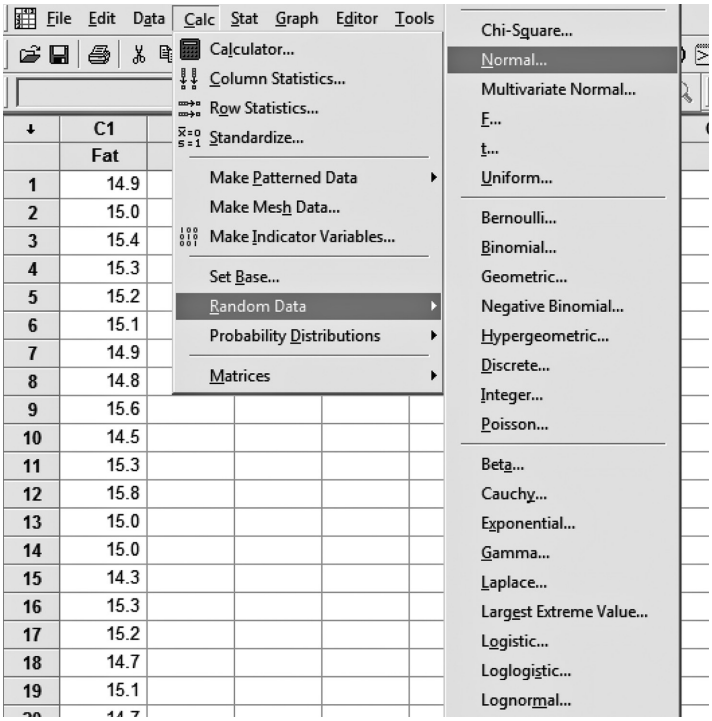


FIGURE 3.18
Selection of "Normal" to generate normally distributed data.

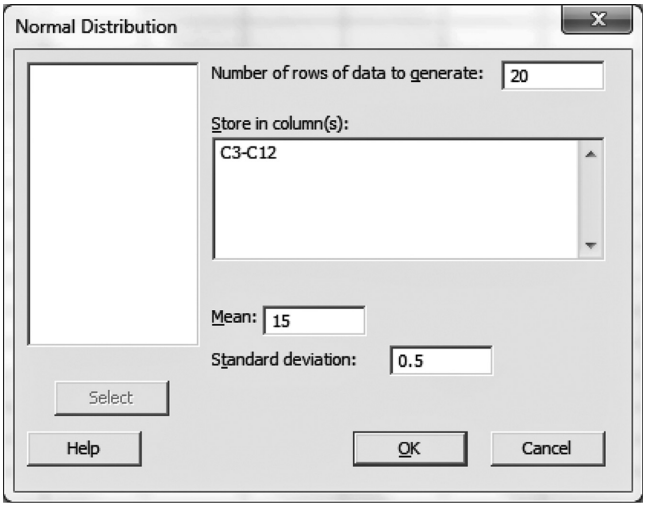
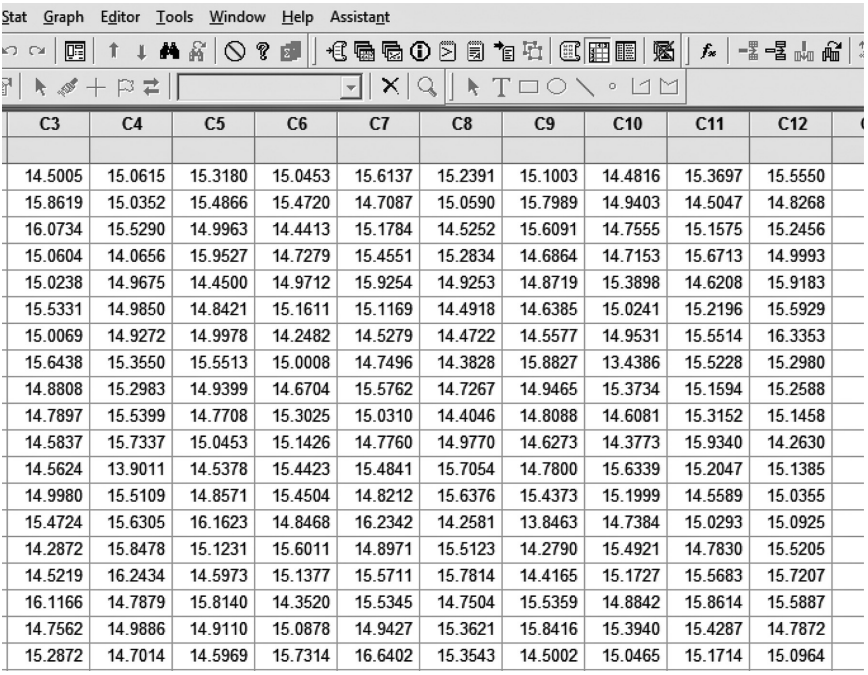


FIGURE 3.19
Mean and standard deviation for generation of random samples.



C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
14.5005	15.0615	15.3180	15.0453	15.6137	15.2391	15.1003	14.4816	15.3697	15.5550
15.8619	15.0352	15.4866	15.4720	14.7087	15.0590	15.7989	14.9403	14.5047	14.8268
16.0734	15.5290	14.9963	14.4413	15.1784	14.5252	15.6091	14.7555	15.1575	15.2456
15.0604	14.0656	15.9527	14.7279	15.4551	15.2834	14.6864	14.7153	15.6713	14.9993
15.0238	14.9675	14.4500	14.9712	15.9254	14.9253	14.8719	15.3898	14.6208	15.9183
15.5331	14.9850	14.8421	15.1611	15.1169	14.4918	14.6385	15.0241	15.2196	15.5929
15.0069	14.9272	14.9978	14.2482	14.5279	14.4722	14.5577	14.9531	15.5514	16.3353
15.6438	15.3550	15.5513	15.0008	14.7496	14.3828	15.8827	13.4386	15.5228	15.2980
14.8808	15.2983	14.9399	14.6704	15.5762	14.7267	14.9465	15.3734	15.1594	15.2588
14.7897	15.5399	14.7708	15.3025	15.0310	14.4046	14.8088	14.6081	15.3152	15.1458
14.5837	15.7337	15.0453	15.1426	14.7760	14.9770	14.6273	14.3773	15.9340	14.2630
14.5624	13.9011	14.5378	15.4423	15.4841	15.7054	14.7800	15.6339	15.2047	15.1385
14.9980	15.5109	14.8571	15.4504	14.8212	15.6376	15.4373	15.1999	14.5589	15.0355
15.4724	15.6305	16.1623	14.8468	16.2342	14.2581	13.8463	14.7384	15.0293	15.0925
14.2872	15.8478	15.1231	15.6011	14.8971	15.5123	14.2790	15.4921	14.7830	15.5205
14.5219	16.2434	14.5973	15.1377	15.5711	15.7814	14.4165	15.1727	15.5683	15.7207
16.1166	14.7879	15.8140	14.3520	15.5345	14.7504	15.5359	14.8842	15.8614	15.5887
14.7562	14.9886	14.9110	15.0878	14.9427	15.3621	15.8416	15.3940	15.4287	14.7872
15.2872	14.7014	14.5969	15.7314	16.6402	15.3543	14.5002	15.0465	15.1714	15.0964

FIGURE 3.20
Ten random samples with sample size of 20.

For simulating the confidence intervals for the 10 samples generated, select "Interval Plot" as shown in Figure 3.24. Doing so opens the dialog box shown in Figure 3.25. Select "Simple" under "Multiple Ys" and click on "OK". That will open the dialog box shown in Figure 3.26. Select "C3-C12" columns for "Graph variables" and click on "OK". The intervals plot (95% confidence intervals) for the 10 samples shown in Figure 3.27 are created. As shown in Figure 3.28, right-click anywhere on the interval plot and select "Reference Lines". This opens the dialog box shown in Figure 3.29. Enter "15" for "Show reference lines at Y values" and click on "OK". This adds the reference line of 15, as shown in Figure 3.30. Because most of the confidence intervals contain the population mean of 15 grams, this simulation confirms that the population mean estimate is indeed 15 grams. If the simulation is performed for a large number of samples, 95% of the confidence intervals are expected to contain the population mean of 15 grams.

The operations manager now wishes to check how many of the customers are dissatisfied after the process has improved. To this end, he randomly selects 1,200 customers who bought hamburgers and finds that 25 of them are dissatisfied with the food quality. He then proceeds to construct a confidence interval for the proportion of all customers who are dissatisfied. In the dialog box shown in Figure 3.31, enter "25" for "Number of events" and

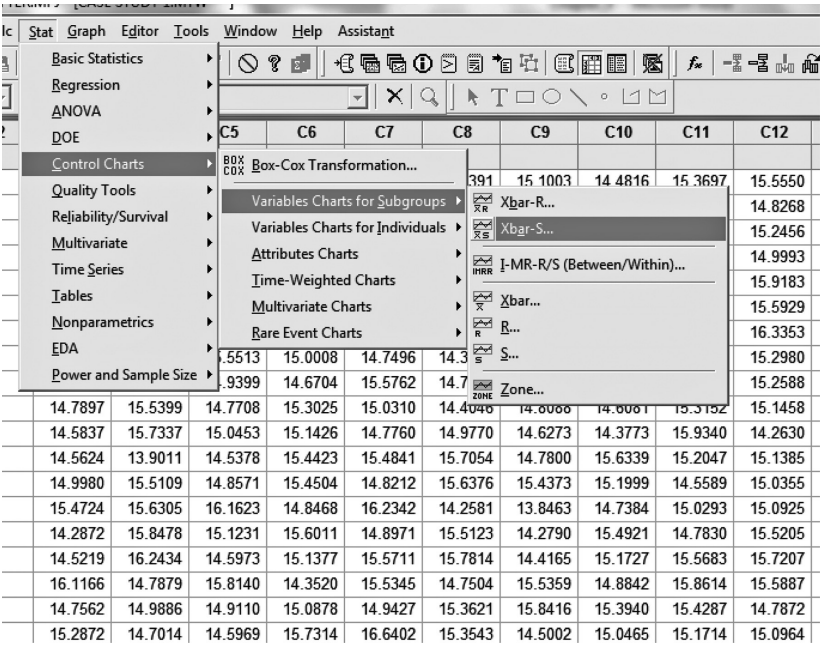


FIGURE 3.21
Selection of "Xbar-S".

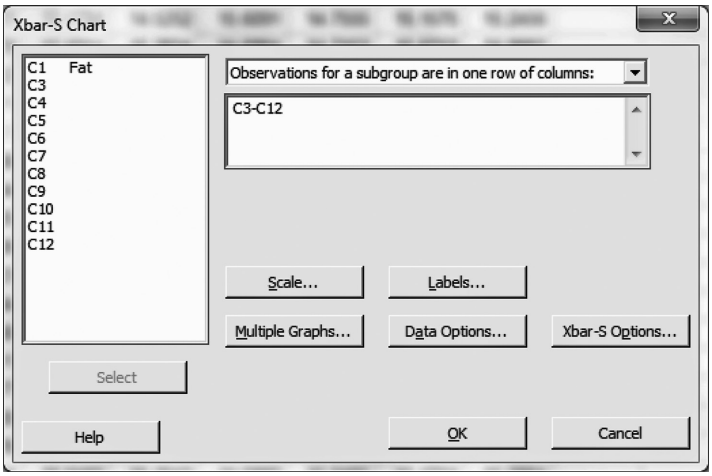


FIGURE 3.22
Selection of variables for Xbar-S charts.

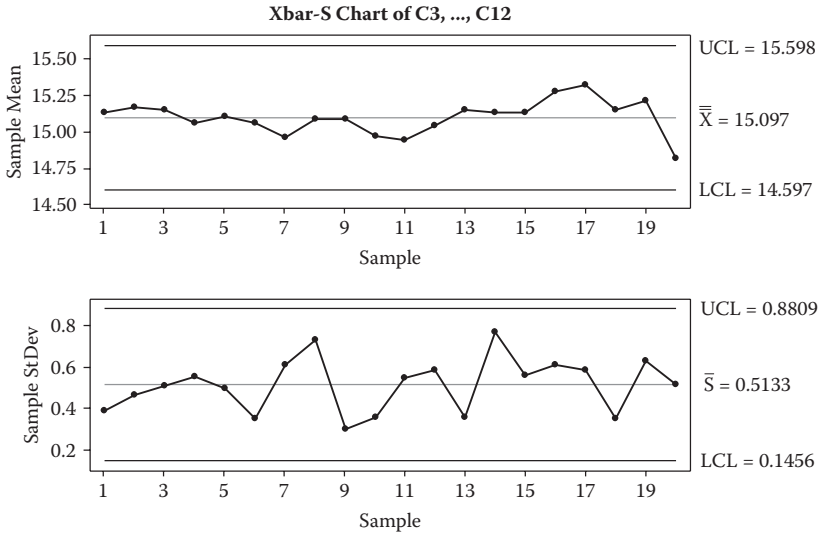


FIGURE 3.23
Xbar-S charts.

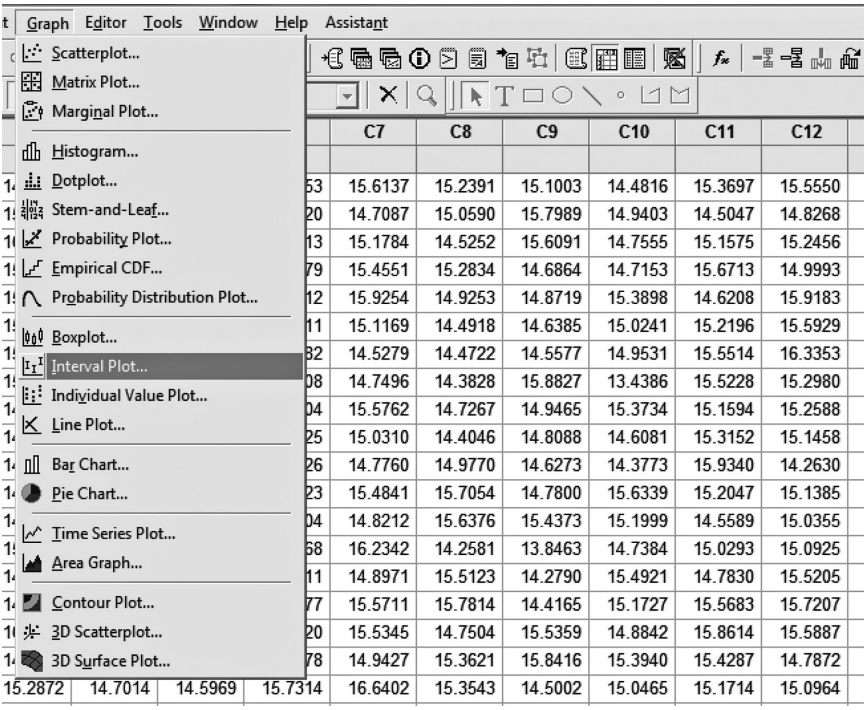


FIGURE 3.24
Selection of "Interval Plot" for simulation of confidence intervals.

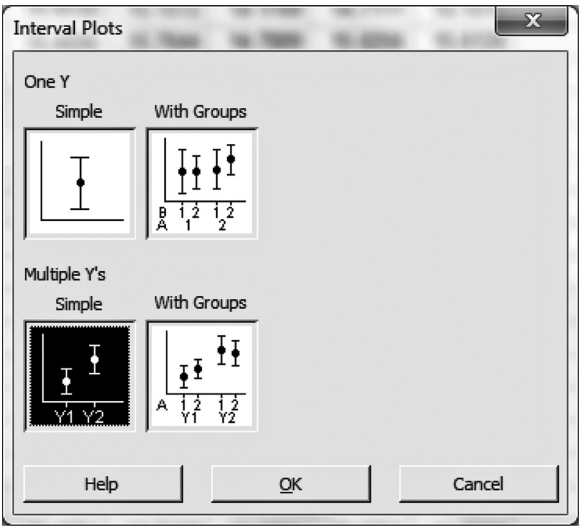


FIGURE 3.25
Selection of "Simple" interval plots under "Multiple Y's".

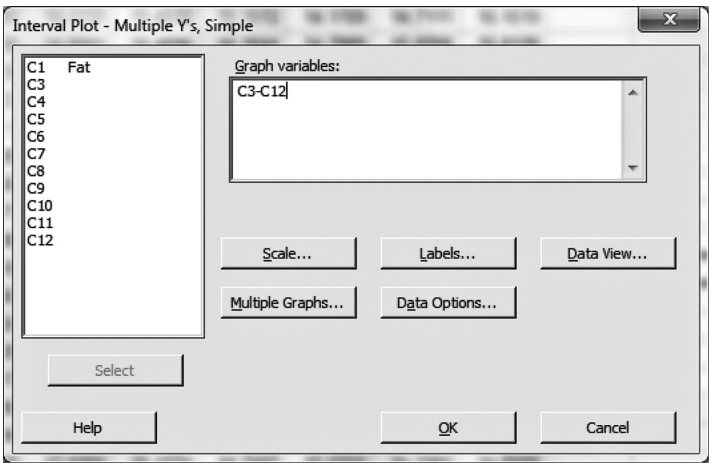


FIGURE 3.26
Selection of variables for simulation.

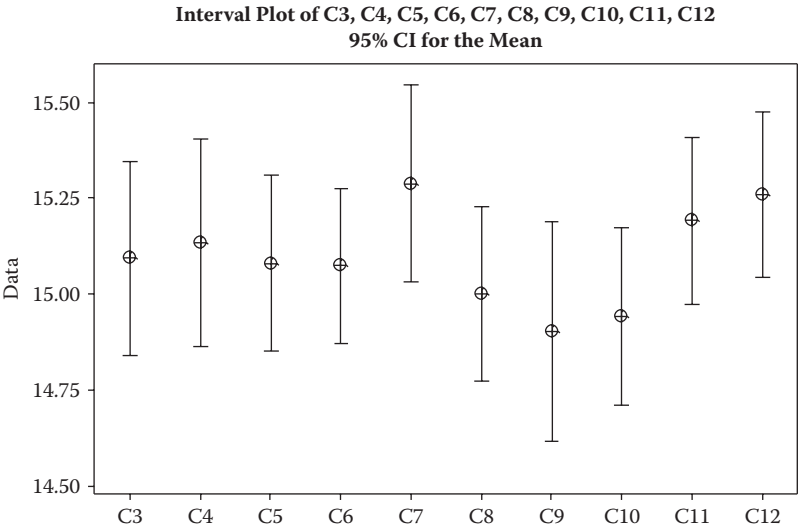


FIGURE 3.27
Interval plot without reference line.

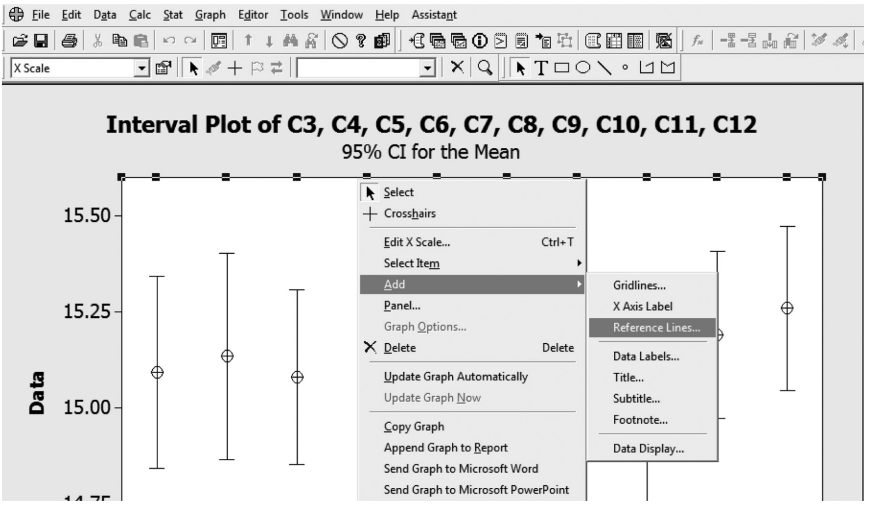


FIGURE 3.28
Process to show reference line.

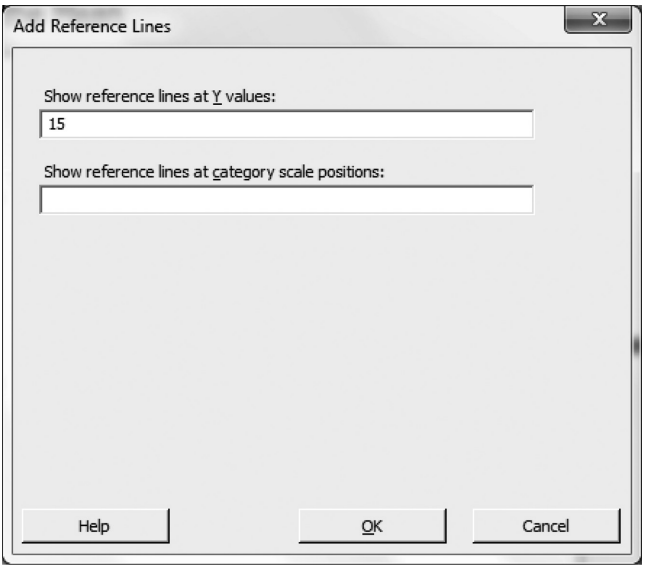


FIGURE 3.29
Entry of reference line.

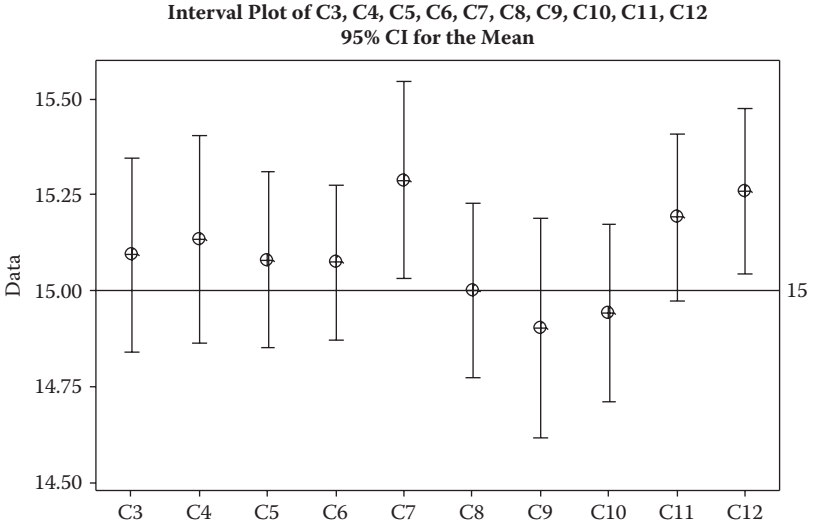


FIGURE 3.30
Interval plot with reference line.

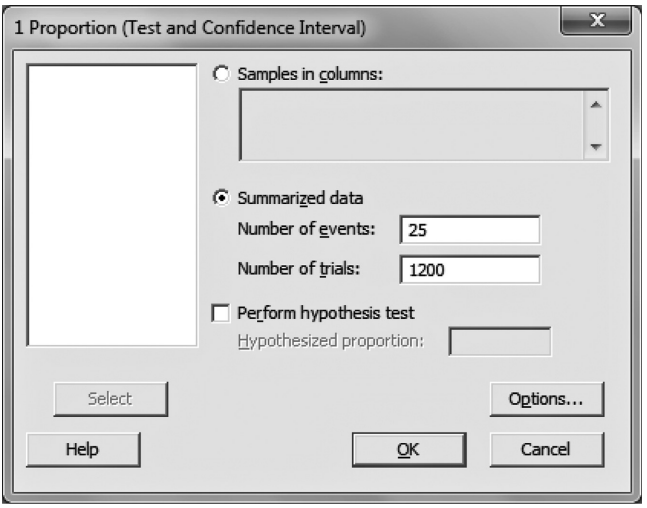


FIGURE 3.31
Entering number of events and trials after improvement.

"1200" for "Number of trials". Click on "Options" and select "less than" for "Alternative" because the operations manager is interested in lowering the proportion of all customers who are dissatisfied. Also, check the box for "Use test and interval based on normal distribution". Click on "OK" and it takes you back to the dialog box shown in Figure 3.31. Click on "OK" and the output shown in Figure 3.32 is the result. It is clear that that there is a 95% probability that the proportion of all customers who are dissatisfied is less than 0.027 (or 2.7%). The operations manager is satisfied that the process has improved.

Test and CI for One Proportion

Sample	X	N	Sample p	95% Upper Bound
1	25	1200	0.020833	0.027615

Using the normal approximation.

FIGURE 3.32
One-sided 95% confidence interval for population proportion after improvement.

3.5 Control Phase

The operations manager plans to check the hamburger preparation process on the three grills regularly, in order to ensure that the customers consistently get what they are promised.